Study on Modified Mixing Model of Temperature-stratified Thermal Storage Tank under Variable Input Condition

Hiroaki KITANO¹, Takeshi IWATA¹, Kazunobu SAGARA²
¹ Department of Architecture, Mie University, 1515 Kamihama, Tsu, Mie, 514-8507, Japan, e-mail: kitano@arch.mie-u.ac.jp, iwata@arch.mie-u.ac.jp
² Department of Architectural Engineering, Graduate school of Engineering, Osaka University, 2-1 Yamadaoka, Suita, Osaka, 565-0871, Japan, e-mail: sagara@arch.mie-u.ac.jp

KEY-WORDS
Stratification, Model, Variable input, Chilled-water

ABSTRACT

A mixing model for temperature-stratified thermal storage tank under steady conditions of input temperature and flow rate had been presented. This model is known as the R-value model and has been used for a basic performance estimation of the thermal storage tank. In this model, the tank is divided into two regions; a perfect mixing region and a piston flow region with one-dimensional diffusion. And the depth of the perfect mixing region has relation to Archimedes number, the inlet geometric conditions and the non-dimensional time. However, this model is not applied to simulation under unsteady input conditions in actual HVAC systems.

In this paper, a mixing model, which can be applied to temperature-stratified thermal storage tank under variable input conditions, is presented, and the temperature profile in tank calculated by using this model is compared with a series of experimental results under variable input conditions.

1. INTRODUCTION

Thermal storage systems are being widely used in Japan and other countries. They contribute to the effective use of energy; peak shift of electrical demand, heat recovery, solar energy utilization and seasonal storage. Water, ice and other phase change materials are used as the thermal storage media. Water thermal storage has a long history in Japan and most of the large scale applications have used water as a storage medium. Type of water storage tank is roughly classified into two type; the temperature-stratified type and the multi-connected complete mixing type. The storage performance of the temperature-stratified type is above the multi-connected complete mixing type. Studies on mixing process in the temperature-stratified tank have been carried out, and a mixing model for the temperature-stratified thermal storage tank under steady input conditions has been presented in earlier publications [1]. This model is used for a basic performance estimation of the storage tank. However, this model is not applied to simulation under unsteady input conditions in actual HVAC systems. A mixing model for variable input conditions was presented in our previous paper [2].

In this paper, a mixing model, which can be applied to temperature-stratified thermal storage tank under variable input conditions, is presented. This model is modified from the mixing model presented in TERASTOCK 2000 [2]. The temperature profile in tank calculated by using this modified model is compared with a series of experimental results under variable input conditions.
2. MIXING MODEL

The mixing model presented in this paper, is based on the mixing model for the temperature-stratified thermal storage tank under steady input conditions [1]. It is also assumed that the storage tank is divided into two regions; a mixing region and a piston flow region with one-dimensional diffusion as shown in figure 1. The difference of the mixing model presented in this paper from our previous models [1][2], is in a mathematical model for the mixing region. It was assumed that water temperature is uniform in the mixing region of our previous models.

The mathematical model for the mixing region and the piston flow region is shown below. Equation (1) is for the mixing region and equation (2) is for the piston flow region with one dimensional diffusion.

\[
\frac{\partial T_{st}}{\partial t} = \frac{\varphi_{(z)}}{A} (T_{in} - T_{st}) - \int_{0}^{z} \frac{\varphi(z)}{A} d\zeta \frac{\partial T_{st}}{\partial z} \quad (0 \leq z \leq l_m) \tag{1}
\]

\[
\frac{\partial T_{st}}{\partial t} = \kappa \frac{\partial^2 T_{st}}{\partial z^2} - U \frac{\partial T_{st}}{\partial z} \quad (l_m < z \leq L) \tag{2}
\]

The depth of the mixing region, \(l_m\), is given as the following experimental equation (equation (3)).

\[
l_m = 0.7 d_o Ar_{in}^{-0.5} + 0.4 L t^* \tag{3}
\]

where the inlet Archimedes number is a dimensionless number and expresses the ratio of buoyancy force to inertia force defined as equation (4).

\[
Ar_{in} = d_o g \frac{\rho_{ref} - \rho_{in}}{\rho_{ref} u_{in}^2} \quad (\rho_{in} < \rho_{ref}) \tag{4}
\]

The reference density of tank water, \(\rho_{ref}\), is set according to the following procedure: It is presupposed that the water jet flows into the tank from the surface of tank water, and water density of the jet is equal to input water density into the tank and velocity of the jet is equal to the one of input water. It is assumed that the vertical velocity of this water jet at the depth \(z\), is calculated from the following equation (5) [3].

\[
\rho_{in} u_{jet(z)}^2 = \rho_{in} u_{in}^2 - \int_{0}^{z} (\rho_{ref} - \rho_{in}) g d\zeta \tag{5}
\]

\[
T_{ref} = T_{st}|_{z=l_b} \tag{6}
\]

The depth \(l_b\), which the vertical velocity of the jet is equal to zero in equation (5), is found by

---

Figure 1 Two regions in temperature-stratified tank for mixing model
substituting the velocity of water jet at surface and water density of the water jet into equation (5). The water temperature at the depth where the jet reaches, is called a reference temperature in the tank (equation (6)), and the reference density in the tank is the density of water at the reference temperature.

The non-dimensional time, $t^*$, in equation (3) is set according to the following procedure: The heat, $Q_{st,o}$, is calculated by equation (7). $Q_{st,o}$ expresses the stored heat in the part where water temperature is higher than the reference temperature in the tank. The time, $t_{in}$, is numerically solved under the condition which the heat, $Q_{in,o}$, calculated by equation (8) is equal to $Q_{st,o}$. And the non-dimensional time is calculated by equation (9).

$$Q_{st,o} = c_p \int_0^{t_1} \left( T_{st} - T_{ref} \right) d z \quad (7)$$

$$Q_{in,o} = c_p \int_{t-t_{in}}^{t} \left( T_{in} - T_{ref} \right) q_{in} \ d \tau \quad (8)$$

$$t^* = \frac{\int_{t-t_{in}}^{t} q_{in} \ d \tau}{V} \quad (9)$$

The relationship between $\phi$ in equation (1) and the input flow rate into the tank, $q_{in}$, is expressed as equation (10). It is assumed in this paper that the function of $\phi$ is expressed as equation (11).

$$q_{in} = \int_0^{t_{in}} \phi \ d z \quad (10)$$

$$\phi = \frac{3}{2} \frac{q_{in}}{l_m} \left\{ 1 - \left( \frac{z}{l_m} \right)^2 \right\} \quad (11)$$

### 3. EXPERIMENTAL VALIDATION

The water volume in the experimental tank was 786 x 786 x 800 mm. The horizontal and vertical section of the experimental tank are shown in figure 2. The vertical distribution of water temperature was measured for 21 points with 40 mm gap in the experimental tank and input water temperature and flow rate was also measured for the interval of 30 seconds. The four cases of experiments which were different in the variation of input flow rate, were carried out. And the results of two experiments are shown in this paper.

These equations are solved by an explicit finite differential method. The tank water is vertically divided into intervals of 4 mm and time increment is 0.1 seconds. In this calculation, the vertical temperature distribution in the storage tank were calculated by using the initial temperature in the tank and the input conditions measured during each experiment.

![Figure 2](image_url)
The variation in input water temperature and input flow rate are shown in figure 3. And the temperature profile for different number of water exchanges defined as \( t_w^* = \frac{\int q_{in} \, dt}{V} \) in the experimental tank is shown in figure 4. Figure 5 shows the results which was calculated by using the mixing model presented in TERASTOCK 2000 [2]. On this experiment, the input water temperature was kept constant after increasing from initial temperature of tank water to about 12 °C in the beginning of the experiment and the input flow rate changes as shown in figure 3. The calculated results for the temperature distribution in figure 4 agree very well with the experimental results. As shown in figure 5, The calculated water temperature of the mixing region dose not agree with the experimental one during the water temperature in the mixing region increasing, because the previous mixing model gives the results which the temperature of the mixing region is uniform. As a result, the position of stratification in temperature profiles does not agree well.
The variation in input water temperature and input flow rate are shown in figure 6. And the temperature profile for different number of water exchanges is shown in figure 7. Figure 8 shows the results calculated by using the mixing model presented in TERASTOCK 2000 [2]. In this experiment, the calculated results for the temperature distribution shown in figure 7, agree well with the experimental results. As shown in figure 8, It is found that the calculated water temperature of previous model was uniform in the mixing region in the beginning of experiment and the position of stratification in temperature profile and the temperature of the mixing region did not agree, and the difference in the position of stratification subsequently remained in temperature profiles during the experiment.
4. CONCLUSIONS

In this paper, a mixing model which can be applied to temperature-stratified thermal storage tank under variable input conditions, is presented, and the temperature profiles in the thermal storage tank calculated by this model were compared with the results of experiments under variable input conditions. It was found that the calculated results by using the modified mixing model agreed well with the experimental results.

Acknowledgement
The investigations described in this paper have been financially supported by the Japanese Society for Promotion of Science.

Nomenclature

\[ A \] : horizontal sectional area of tank  \([-]\)
\[ AR_{in} \] : inlet Archimedes number  \([-]\)
\[ d_0 \] : diameter of inlet pipe  \([m]\)
\[ l_m \] : depth of mixing region  \([m]\)
\[ l_b \] : depth from water surface where the jet reaches  \([m]\)
\[ L \] : total depth of water in thermal storage tank  \([m]\)
\[ g \] : gravity acceleration  \([m/s^2]\)
\[ q_{in} \] : input flow rate  \([m^3/s]\)
\[ R \] : non dimensional depth of mixing region, \( (R = l_m / L) \) \([-]\)
\[ R_0 \] : variable of model  \([-]\)
\[ R_k \] : constant of model  \([-]\)
\[ t, \tau \] : time  \([sec]\)
\[ t^* \] : non-dimensional time  \([-]\)
\[ T_{st} \] : water temperature in thermal storage tank  \([^\circ C]\)
\[ T_{in} \] : temperature of input water  \([^\circ C]\)
\[ T_{ref} \] : reference temperature in thermal storage tank  \([^\circ C]\)
\[ u_{in} \] : velocity of input water  \([m/s]\)
\[ u_{jet}(z) \] : vertical velocity of the water jet at the depth \(z \)  \([m]\) in tank  \([m/s]\)
\[ U \] : velocity at cross section of tank  \([m/s]\)
\[ V \] : water volume in thermal storage tank  \([m^3]\)
\[ z, \zeta \] : depth from water surface in thermal storage tank  \([m]\)
\[ \kappa_0 \] : thermal diffusivity of water in thermal storage tank  \([m^2/s]\)
\[ \varphi \] : variable of model  \([m^2/s]\)
\[ \rho_{im} \] : density of input water  \([kg/m^3]\)
\[ \rho_{ref} \] : reference density of water in tank at the reference temperature \( \rho_{ref} = f( T_{ref} ) \)  \([kg/m^3]\)
\[ \rho_{st}(z) \] : density of water at the depth \(z \)  \([m]\) in thermal storage tank  \([kg/m^3]\)

References