1. Introduction
In this article an unsteady-state two dimensional simulation model of the solar chimney will be introduced. As previous results [1, 2, 3] showed, it is not possible to predict flow rates of solar chimneys with simple models in all cases. A one-dimensional model cannot take the flow pattern inside the channel into account, thus over-predicting the flow rate for thick and vertical chimneys. Experiments [1] proved that a solar chimney needs a long time to reach thermal equilibrium. This can take 4-5 hours even for chimneys built from light materials. Thus a real solar chimney never operates at thermal equilibrium; as environmental temperature and solar radiation change, the chimney warms up and cools down periodically. An improved prediction model is essential to calculate flow rate even for this unsteady case.

2. Simulation model
A multi-zonal model can be constructed by dividing the chimney into a great number of blocks from the bottom to the top. In order to simulate the flow pattern, fluid in the channel is further divided into layers parallel to the wall. Wall, glazing and fluid temperatures, heat transfer coefficients are calculated at each block. The backside structure of the chimney with the insulation is divided into wall layers parallel to the wall and unsteady state heat transfer is calculated between each element, the flowing fluid in the chimney and the environment.

Then the number of air layers taking part in heat transfer is calculated with the boundary layer thickness equation introduced in a previous paper [4]. Then the flow rate is calculated using ventilation rate equations for the given temperature distribution.

2.1 Unsteady state heat dissipation in the chimney wall structure
Figure 2 shows the general alignment of a wall structure element (block i and wall layer k) and the heat transfers considered.

As can be seen, there are 4 heat transfers possible, and the net energy balance of these heat transfers will result the temperature increase of the element.

Energy equation for elemental time:

\[
\frac{\Delta T_{(i,k)}}{\Delta t} = \left( \frac{q_{w,i}A_r}{\Delta x} - T_{(i,k)} \right) \frac{A_r k_{(i,k)}}{\Delta y} + \left( \frac{q_{w,i}A_r}{\Delta y} - T_{(i,k)} \right) \frac{A_r k_{(i,k)}}{\Delta x} + \frac{T_{(i-1,k)} - T_{(i,k)}}{\Delta x} \frac{A_i k_{(i-1,k)}}{\Delta x} + \frac{T_{(i+1,k)} - T_{(i,k)}}{\Delta x} \frac{A_i k_{(i+1,k)}}{\Delta x}
\]

**Figure 2. General element**

If the element is on the side or bottom of the structure, one equation changes to a convection heat transfer equation. If the element is the wall element, the equation will have the following form:

\[
M_c A_r \frac{\Delta T_{w(i)}}{\Delta t} = q_{w,i} A_r + (T_{(i,k)} - T_{w(i)}) A_r k_{(i,k)} + \frac{T_{(i-1,k)} - T_{w(i)}}{\Delta x} A_i k_{(i-1,k)} + \frac{T_{(i+1,k)} - T_{w(i)}}{\Delta x} A_i k_{(i+1,k)}
\]

There is a new part representing radiation heat loss towards the glazing, \( R_{w-g} = \frac{1 - \varepsilon_r}{\varepsilon_r w} \cdot \frac{1 - \varepsilon_g}{\varepsilon_g A_g} \)

Ventilation Performance of Solar Chimneys
Part 7. Unsteady state simulation model for solar chimneys

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2.2 Heat transfer at the glazing
Glazing has minimal temperature distribution and simulating this distribution gives a great number of radiation heat transfer equations difficult to handle. That is why an average glazing temperature was used. Glazing has gain from the absorbed solar radiation and from the radiation of the wall. It loses heat by convection and radiation to the environment.

The balance equation is:

\[ c_{pl,s} \frac{\Delta T_s}{\Delta t} = wL \left( \sum_{n=1}^{\infty} q_{\omega(n)} + q_{\omega} + q_{\alpha} + q_{\sigma} \right) \]

Parts of the equation:
- Energy loss by radiation:
  \[ q_{\sigma} = \frac{T_s^4 - T_e^4}{R_{\sigma} A_x} \]
  \( R_{\sigma} = \frac{1}{\varepsilon_x A_x} + \frac{1}{\varepsilon_x A_x A_{\sigma} F_{\sigma}} \)
  \( A_x = wL \), and \( F_{\sigma} = 1 \).
- Energy gain from incident solar radiation:
  \( q_{\sigma} = h_x \left( T_s - T_i \right) \)
- Energy gain from a block of wall by radiation:
  \[ q_{\omega(n)} = \frac{T_{\omega(n)}^4 - T_e^4}{R_{\omega} A_x} \]
  \( R_{\omega} \) can be calculated as:
  \[ R_{\omega} = \frac{1}{\varepsilon_x A_x} + \frac{1}{\varepsilon_x A_x A_{\omega} F_{\omega}} + \frac{1}{\varepsilon_x A_x} \]
  \( A_{\omega} = wL/n \), and it can be assumed that \( F_{\omega} = 1 \), as the area of the glazing is huge compared to the area of a wall block.

2.3 Convective heat transfer coefficients
Based on the experimental data and available literature, the following heat transfer coefficient is used at the wall [5]:

In the laminar region (\( Gr_{(i)} < 10^9 \)):

\[ h_{\omega(i)} = \frac{k}{l (L/n)} 0.59 \sqrt{Gr_{(i)} Pr} \]

In the turbulent region (\( Gr_{(i)} > 10^9 \)):

\[ h_{\omega(i)} = \frac{k}{l (L/n)} 0.1 \sqrt{Gr_{(i)} Pr} \]

2.4 Heat transfer equations for the flowing fluid
If uniform dissipation of the convected heat is assumed, the following differential equation can be set up for the flowing fluid:

\[ \frac{mc_p}{\Delta x} \frac{dT_f}{dx} = h_f (T_w - T_a) \]

where \( T_a, T_w, h_f \) are fluid and wall temperature, and wall heat transfer coefficient. As the fluid in the channel is divided into layers, the number of layers involved in heat transfer must be calculated. A suitable equation for the boundary layer thickness is needed. However, it is difficult to find an equation that contains all the variables influencing the boundary thickness and used in the model. That is why the following equation is introduced [4]:

\[ \delta_{(i)} = \frac{0.58}{f_i^{1/2} 0.65 \sin^{1/2} \theta} \left( \frac{1 + 0.494 P_r^{1/10}}{Gr_{(i)}} \right)^{1/10} \]

The differential equation for fluid heat balance must be changed to suit the multi-zonal model. Air in the channel is divided into layers in each block. See Figure 4 for details of the division. The number of layers taking part in the heat transfer must be decided first with the boundary layer thickness equation. There are two cases, depending on the number of layers involved:

If the thermal boundary does not reach to a new layer in the given block, the following equation can be used to calculate the temperatures in layer (1) for a given block (i):

\[ T_f(i) = \frac{T_{f(i-1)} m_c p_{a(o)} + T_{w(i)} h_i \omega(i)}{m_c p_{a(o)} + h_i \omega(i) \frac{wL}{n}} \]

Figure 4. Model division

Other layers are not involved in the heat transfer, and their temperature remains inlet temperature. If there is a new layer (2) that is inside the thermal boundary of the flow, the following form should be used:

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where \( m_1, m_2 \) is the mass flow inside the boundary in the previous block, and the mass flow of the layer that joins the heat transfer in the recent one. All the layers inside the boundary will have the same fluid temperature from that block on.

2.5 Flow rate calculation

When calculating the flow rate of the chimney, ventilation flow rate for each layer is calculated separately and then summarized to get the overall flow rate of the chimney. Discharge coefficients for the whole channel are used for the calculation of individual layers.

Densities in layer (1):
- **Average**: \( \rho_f = 353.25/\overline{T_f} \)
- **Outlet**: \( \rho_{f(out)} = 353.25/\overline{T_{f(out)}} \)

Volume flow rate equations for layer (1):
- At the outlet:
  \[ Q_m = C_{m1}A_1 \frac{-2}{\rho_{f(out)}} \left[ (\rho_e - \rho_f)g \sin \varphi + p_m \right] \]
- At the inlet:
  \[ Q_0 = C_{m1}A_1 \frac{2}{\rho_e} (-p_m) \]

The overall flow rate of the chimney is calculated by simply summing the individual layer flow rates.

3. Results

3.1 Simulation and experimental setup

The experiments were carried out with the solar chimney apparatus described in a previous paper [4].

![Figure 5. Simulated measurement points](image)

Values of variables and settings are summarized in Table 1. The simulation setup was coordinated with the experiments.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclination (( \varphi ))</td>
<td>60°</td>
</tr>
<tr>
<td>Channel thickness (( s ))</td>
<td>20 cm</td>
</tr>
<tr>
<td>Channel length (( L ))</td>
<td>2 m</td>
</tr>
<tr>
<td>Opening area (( f ))</td>
<td>100%</td>
</tr>
<tr>
<td>Absorbed heat flux (( q_{abs} ))</td>
<td>300 W/m²</td>
</tr>
<tr>
<td>Inlet discharge coefficient (( C_{in} ))</td>
<td>0.8*</td>
</tr>
<tr>
<td>Outlet discharge coefficient (( C_{out} ))</td>
<td>2.1*</td>
</tr>
<tr>
<td>Environment temperature (( T_e ))</td>
<td>21 °C</td>
</tr>
</tbody>
</table>

*dynamic pressure is assumed to be remaining in channel. Channel friction loss considered.

There were 5 wall layers, 3 air layers and 20 blocks in the simulation model. The measurement points that were simulated can be seen on Figure 5. Styrene foam measurement points were located on both sides of the foam, so the actual plots are average temperatures for the experiments. Flow rate measurement points were discussed in a previous paper [4].

3.2 Discussion

As it can be seen on Figure 6, wall measurement points follow the same heat up pattern for both the experimental and simulation case. But the resulting temperatures are not all agreeing. There is a 4 degrees disagreement at the P10 point, but this difference is still fairly small.

![Figure 6. Wall temperatures](image)

Figure 7 shows the insulation temperatures. The S1 point at the inlet has a good agreement, while the other two points are not agreeing. There is a 5 degrees difference. A possible cause for this difference is that insulation was made of panels and these two points are close to the edge of the panels, thus heat bridges could have occurred. Also, thermal properties could have been different than the input data of the simulation.

![Figure 7. Temperatures inside the insulation](image)

Temperatures for the three air layers (1, 2 and 3) are plotted. The layer closer to the wall had higher air temperature. The plot of mass flow rate is shown on Figure 9. When the thermal equilibrium was reached, the calculated mass flow rate was similar to the mass flow rate measured. This is also plotted on the figure.
4. Conclusion

The unsteady state two-dimensional simulation model proved to be very useful for performance prediction. Both wall temperatures and volume flow rates were matching with the experimental data. The small differences found can be minimized using more correct material properties. In the future a model that considers geographical location and real time solar radiation should be set up. Also, accuracy of the model can be increased with more layers as the recent simulation with only three layers introduces a big assumption into calculation. The boundary layer thickness equation can also be improved using the experimental data taken at the experiments.

5. Acknowledgement

The authors would like to thank to Ms. Yoriko Murayama at INAX Corporation for her valuable contribution to this research.

6. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>length of the chimney (m)</td>
<td></td>
</tr>
<tr>
<td>s</td>
<td>chimney thickness (m)</td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>distance from inlet (m)</td>
<td></td>
</tr>
<tr>
<td>Φ</td>
<td>inclination of the chimney (°)</td>
<td></td>
</tr>
<tr>
<td>w</td>
<td>width of the chimney (m)</td>
<td></td>
</tr>
<tr>
<td>C_d, C_w</td>
<td>inlet and outlet discharge coefficient (-)</td>
<td></td>
</tr>
<tr>
<td>A_i, A_o</td>
<td>inlet and outlet opening area (m²)</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>number of blocks in the model</td>
<td></td>
</tr>
<tr>
<td>j, p</td>
<td>number of air and wall layers</td>
<td></td>
</tr>
<tr>
<td>f_i</td>
<td>inlet opening ratio (A_i/A_o) (-)</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>radiation shape factor (-)</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>element mass (kg)</td>
<td></td>
</tr>
<tr>
<td>Δt</td>
<td>time increment</td>
<td></td>
</tr>
<tr>
<td>c_p</td>
<td>specific heat (J/kgK)</td>
<td></td>
</tr>
<tr>
<td>ν</td>
<td>dynamic viscosity of air (m²/s)</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>heat conductivity (W/mK)</td>
<td></td>
</tr>
<tr>
<td>β</td>
<td>volume coefficient of expansion (1/K)</td>
<td></td>
</tr>
<tr>
<td>α, ε, τ</td>
<td>absorptivity, emissivity, transmissivity (-)</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>temperature (K)</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>heat transfer coefficient (W/m²K)</td>
<td></td>
</tr>
<tr>
<td>q</td>
<td>heat flux (W/m²)</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>mass flow rate (kg/s)</td>
<td></td>
</tr>
<tr>
<td>σ</td>
<td>Stefan-Boltzmann constant (W/m²K⁴)</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>acceleration of gravity (m/s²)</td>
<td></td>
</tr>
<tr>
<td>ρ</td>
<td>density (kg/m³)</td>
<td></td>
</tr>
<tr>
<td>ΔT</td>
<td>temperature increase (K)</td>
<td></td>
</tr>
<tr>
<td>ρ_w</td>
<td>ambient pressure at chimney inlet (Pa)</td>
<td></td>
</tr>
</tbody>
</table>

Subscripts:

s, r, c, ed, a, v, h, w, g, f, e

Absorption, conduction, convection, solar, radiation, absorption, vertical and horizontal extent, wall, glassing, fluid and environment, to wall (block and layer).

7. References


*大阪大学大学院工学研究科建築工学専攻
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